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Original Article

Autocorrelation of First 4+ Nuclear Energy States

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ABSTRACT

Objectives: To investigate autocorrelation patterns in the first 4^+ excitation energies of 466 even-even nuclei (A = 10–256) classified by multiple nuclear structure parameters (P-factor, Np·Nn product, $R_4/_2$ ratio, β_2 deformation, and neutron number N).

Material and Methods: Autocorrelation function (ACF) analysis applied to 466 even-even nuclei from the NNDC database. Nuclei sorted by five structural classifiers; ACF and correlation time τ_n computed for each classification scheme using the Sprott (2003) time-series methodology.

Results: The P-factor classification yields the highest ACF value (0.432) and the longest correlation time (τ_n = 74), significantly outperforming all other classifiers. The Np·Nn product, $R_4/_2$ ratio, β_2 deformation, and neutron number N produce progressively lower ACF values and shorter correlation times.

Conclusion: The P-factor is the most effective single parameter for grouping nuclei with similar 4^+ excitation energies, reflecting its superior sensitivity to proton-neutron valence interactions. This result extends the utility of P-factor systematics from the 2^+ state to the 4^+ state in even-even nuclei.

Keywords: Autocorrelation function, Even-even nuclei, Proton-neutron interaction, Theoretical study

INTRODUCTION

The theoretical description of physical systems often distinguishes between integrable and chaotic systems. Integrable systems, which can be solved exactly using linear equations, contrast with chaotic systems, whose nonlinear dynamics lead to complex, unpredictable behavior.[1-5] Random matrix theory (RMT) has been proposed to describe quantum systems approaching full classical chaos.[6]

Nuclear systems often display behaviors that bridge regularity (Poisson-like statistics) and disorder (Wigner-like distributions).[7-16] Identifying reliable indicators for collective nuclear motion remains a critical research goal. Potential markers include atomic number (Z), neutron number (N), mass number (A), the $R_{4/2}$ ratio, P-factor, and deformation parameter (β_2).

Here, we aim to determine the optimal indicator for characterizing nuclear collectivity. Focusing on even-even nuclei for their simplicity and data availability, this work extends prior research on 2^+ states[16] to include 4^+ excitations. Data for the first 4^+ levels were sourced from the National Nuclear Data Center (www.nndc.bnl.gov), covering 466 even-even nuclei from A = 10 to 256.

Excluding shell-closure influences, lighter nuclei typically show 4^+ excitations in the (MeV) range, whereas heavier ones drop to hundreds of (keV). Thus, these 4^+ energies serve as proxies for collectivity trends. We apply autocorrelation analysis to uncover patterns in these datasets.

MATERIAL & METHODS

In this analysis, the sequences of 4^+ energies, organized by various structural parameters, are treated as discrete time series. The autocorrelation function (ACF) is then applied to measure the internal correlation within each series as a function of the lag, k. For an uncorrelated series, approximately 95% of the ACF values for different k lags are expected to fall within a confidence band of ±2 standard deviations around zero.[17] The ACF has a value of 1 at k=0 and typically decays toward zero as lags increase. The correlation time, τ_c, is defined as the lag at which the ACF drops to 1/e (approximately 0.37) of its initial value.[17] A slower decay, indicated by a higher τ_c, is interpreted as a signature of stronger, longer-range correlations associated with collective behavior. The ACF formula is:

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$$ACF = \frac{\sum_{i=1}^{N-k}\left(E_i - \bar{E}\right)\left(E_{i+k} - \bar{E}\right)}{\sum_{i=1}^{N}(E_i - \bar{E})^2}, \tag{1}$$

where N=466 is the sample size, E_i is the energy of the i-th nucleus, and Ē is the mean energy. To ensure reliability, k is limited to N/4.[17] Given the broad energy span (keV to MeV) across A=10–256, we analyze the natural logarithm of energies to normalize variability.

Classifying parameters

Z, N, and A parameters

These quantities underpin residual proton-neutron interactions, which are crucial for the development of collectivity. Notable shifts in nuclear traits occur at the magic numbers 2, 8, 20, 28, 50, 82, and 126.

β_2 *parameter*

In the Bohr-Mottelson collective framework,[18,19] the radius of axially symmetric nuclei is:

$$R(\theta,\varphi) = R_{av}\left[1 + \beta_2 Y_{20}(\theta,\varphi)\right], \tag{2}$$

β_2 quantifies deviation from sphericity, with positive/negative values denoting prolate/oblate shapes, aiding collectivity assessment.

$R_{4/2}$ *ratio*

The $R_{4/2}$ ratio, defined as $E(4_1^+)/E(2_1^+)$, is a well-established empirical measure of nuclear collectivity, with its utility grounded in both experimental and theoretical foundations.[20,21]

Experimentally, the evolution of this ratio tracks the structural transition of a nucleus. As a nucleus evolves from a spherical vibrator to a deformed rotor, the energies of the 2_1^+ and 4_1^+ states systematically decrease. Consequently, the $R_{4/2}$ ratio increases from values near 1.2–1.6 for magic nuclei (indicating a lack of collectivity) to approximately 2.0–2.2 for vibrational nuclei. For nuclei in a transitional region, the ratio falls between 2.5 and 3.0, finally approaching the idealized value of 3.33 for well-deformed rotational nuclei.[20] A distinct linear correlation between $E(4_1^+)$ and $E(2_1^+)$ levels has also been observed in pre-collective nuclei, which are characterized by an $R_{4/2}$ ratio below 2.[21]

Theoretically, the Interacting Boson Model (IBM) provides a robust framework that reproduces these characteristic values through its three dynamical symmetries.[22] The U(5) symmetry, describing vibrational nuclei, yields an $R_{4/2}$ of 2.00. The SU(3) limit, representing well-deformed rotors, gives the value 3.33, and the O(6) symmetry for γ-unstable nuclei results in a ratio of 2.50. Since most nuclei are transitional and do not conform to these ideal limits, critical point symmetries such as E(5) and X(5) were introduced to describe the sharp phase transitions between them.[23-25] A key finding is that as the IBM Hamiltonian parameters approach any of these three dynamical symmetries, the system becomes more orderly, and indicators of quantum chaos are significantly suppressed.

P-factor

The interactions between valence protons and neutrons primarily govern a nucleus's collectivity and deformation.[26-28] A key measure of the strength of this proton-neutron interaction is the P-factor, which is defined by the following relation:

$$P = \frac{N_n N_p}{N_n + N_p}, \tag{3}$$

In this expression, N_p and N_n denote the counts of valence protons and neutrons. A larger P-factor signifies a more robust p-n interaction, which in turn promotes greater collective behavior within the nucleus.[29-31]

RESULTS AND DISCUSSION

The autocorrelation function (ACF) was computed for the natural logarithm of the first 4^+ energy levels across the 466 even-even nuclei, with the results for each classifying parameter displayed in Figures 1(a-f). Statistical significance, indicated by the 95% confidence level (dotted lines), reveals distinct behaviors. For the Z, N, and A parameters, the ACF decays rapidly, indicating only short-range correlations likely influenced by shell closure effects. A fluctuating, periodic pattern in these ACFs may further reflect the impact of magic numbers. In contrast, classification by the β_2, $R_{4/2}$, and P factors produces ACFs with long-range correlations, evidenced by significantly longer correlation times (τ_c).

Among all parameters, the P-factor yielded the highest ACF value (0.432) and the longest correlation time ($\tau_c = 74$). This result underscores the dominant role of the residual valence proton-neutron interaction in driving collective behavior. Consequently, the P-factor emerges as the most effective indicator for classifying nuclear collectivity, outperforming the other parameters considered in this study.

It is useful to compare the current study with our previous work [17] using Table 1 to present the findings succinctly. The aforementioned analysis reveals a consistent pattern of increasing ACF values as we proceed from Z to A, N, β_2, $R_{4/2}$, and P, suggesting the generalizability of our findings in selecting the P-factor as a suitable candidate for classifying nuclear collectivity in even-even nuclei.

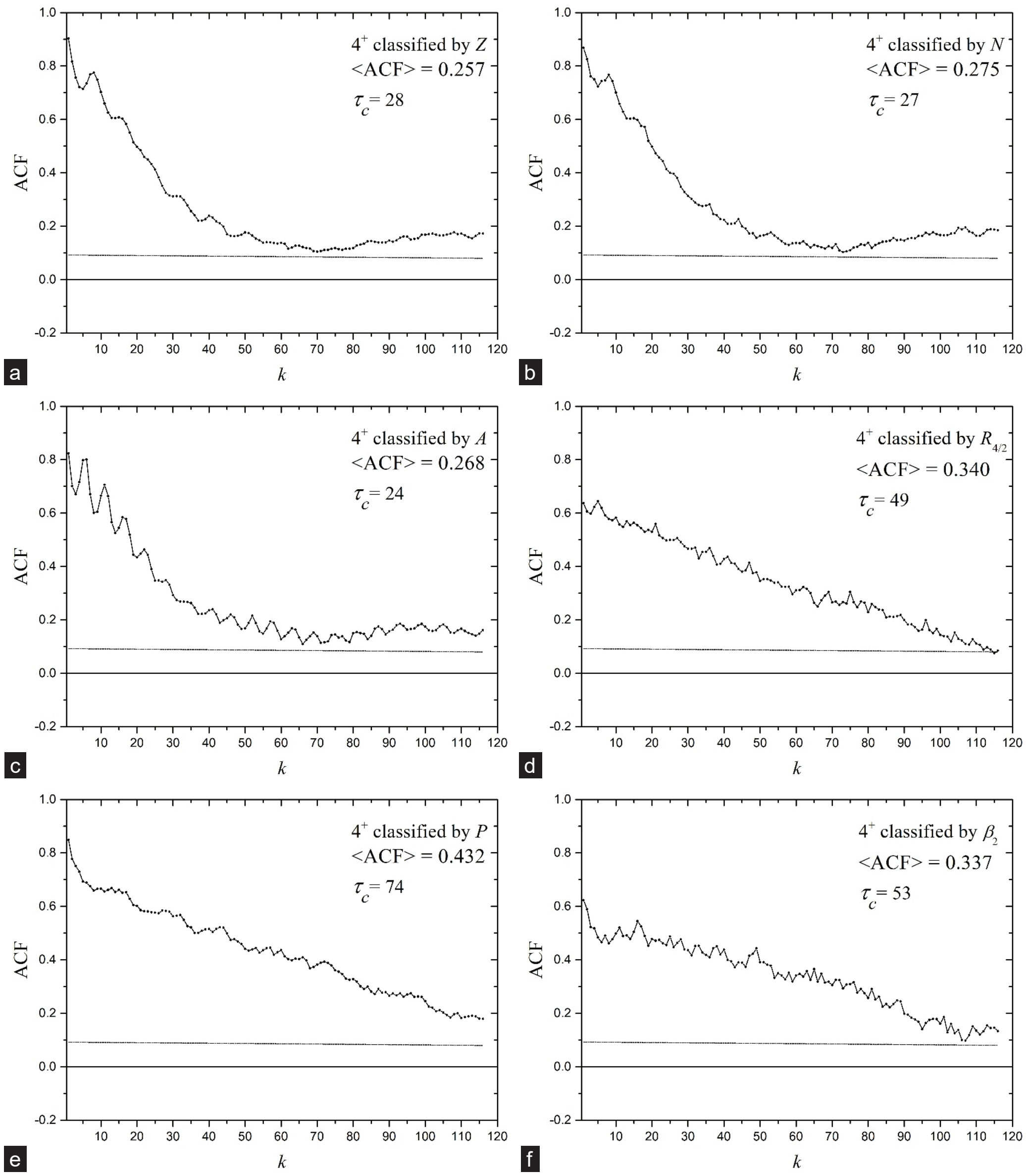

Figure 1: ACF versus lag number k for the first 4^+ states, classified by parameters (a) Z, (b) N, (c) A, (d) β_2, (e) $R_{4/2}$, and (f) P. The correlation time τ_c is indicated. The dotted line shows the 95% confidence level. ACF: Autocorrelation function

Table 1: A comparison between the ACF and the correlation time (τ_c) of the current study and the previous work.

Classifying parameter	2^+ states (Ref. 17)		4^+ states (current study)	
	ACF	τ_c	ACF	τ_c
Z	0.208	16	0.257	28
A	0.208	20	0.268	24
N	0.214	24	0.275	27
β_2	0.391	66	0.337	53
$R_{4/2}$	0.426	64	0.340	49
P	0.471	74	0.432	74

CONCLUSION

This study employed autocorrelation analysis to investigate collective behavior in even-even nuclei. A dataset of the first 4^+ energy levels for 466 nuclei was classified according to several nuclear structure parameters: Z, N, A, β_2, $R_{4/2}$, and the P-factor. A logarithmic transformation was applied to the energy values to stabilize variance across the series before calculating the ACF with a lag number of k = N/4.

The results demonstrate that the P-factor yields the largest ACF enhancement and the longest correlation time (τ_c), underscoring its superiority for identifying collective trends. This finding confirms that the P-factor is an effective parameter for grouping nuclei with similar collective properties. Furthermore, it successfully generalizes the conclusions of our previous work on 2^+ states to the higher 4^+ energy state, reinforcing the robustness of this approach.

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